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MODELING AND SIMULATION OF THE SUCTION PROCESS IN A MULTI-CYLINDER AUTOMOTIVE COMPRESSOR

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ABSTRACT

Thermodynamics, valve dynamics and piston kinematics are considered in order to model the gas flows through compressor suction valves. The equations for the compressor processes based on the first law of thermodynamics are derived to calculate the instantaneous cylinder pressure and temperature. During the suction process, the dynamic motions of an automatic valve between a stop and seat are approximated as one-dimensional in the transverse direction and a linear elastic beam model is used to describe those motions. The finite difference method is then used to solve the equation of the valve. The mass flow rate through the valve is estimated assuming one-dimensional compressible flow through an orifice. All of the equations that are derived are then solved simultaneously to obtain the pressure in the cylinder, valve response and the mass flow rate. Using the calculated mass flow rate, pressure pulsations in a simplified cylindrical annular cavity with an area change to consider 'mode splitting' are predicted based on the characteristic cylinder method. Linear acoustic plane wave theory and a four pole parameter formulation are used to derive and solve the governing inhomogeneous equation for the forced pressure response in the manifold. It is shown that the estimate of the gas pulsation in the suction manifold is in good agreement with experimental results.

1. INTRODUCTION

Reciprocating compressors have been used in industry longer than any other type of machinery. In fact, the compressor is one of the most important elements in any refrigeration unit. Modeling and simulation of the dynamics of compressors are performed so that the effects of various competing design parameters and operating conditions can be assessed without the need for excessive fabrication and testing. Although several modeling techniques have been applied by researchers in the literature to estimate the thermodynamic behaviors of compressors, improvements in these techniques are needed because of the complexities of the flow through the valves and the variability in the flow/refrigerant state throughout the thermodynamic cycle. This work attempts to make these improvements by developing a set of simulation models. Also, gas pulsations are complicated in multi-cylinder compressor suction and discharge manifolds because of the interactions between different cylinder valves. Because the pressure pulsations in the suction manifold are in the range of 1~2% of the static pressure in this research, the gas pulsation can be studied by applying linear acoustical theory.

In this paper, mathematical models have been developed to simulate the compression cycle, valve dynamics and mass flow rate into the compressor cylinder. On the basis of the first law of thermodynamics and a simplified fourth order Bernoulli-Euler linear differential beam equation of the valve, the pressure in a cylinder and resultant pressure pulsations in the suction manifold are predicted. The unsteady flows of a multi-cylinder compressor in the annular suction manifold with an area change, which creates 'mode splitting,' are analyzed and predicted. This research presents a comprehensive simulation model of a reciprocating compressor that explains many of its most important dynamic behaviors.

2. MATHMATICAL MODELING

A configuration of a seven-cylinder reciprocating compressor is illustrated schematically in Figure 1. It is assumed that all of the cylinders are identically made and equally spaced in the circumferential direction. This assumption implies that cylinders are operated in the same manner; thus, one cylinder can be used to represent all cylinders but the mass flow rate, temperature and pressure in other cylinders are delayed through some appropriate time or phase angle. This method is called the characteristic cylinder method (Yang, 1983, Zhow and Hamilton, 1986). First, the model is derived from the first law of thermodynamics. The pressure and temperature response in the cylinder can be calculated using this model. Second, the mass flow rate through the valve is estimated assuming one-dimensional compressible flow through an orifice. The reed valve is modeled with a nonlinear elastic restoring force spring using the finite difference method. In order to investigate the pressure pulsations in the suction manifold, the Heaviside unit step function is used to mathematically describe the area change, which creates acoustic mode splitting. The derivations of the equations are given in the following sections.

2.1 Thermodynamics

The thermodynamic processes in the cylinder govern the performance and efficiency of a reciprocating compressor. The first law of thermodynamics is used here to calculate the instantaneous cylinder pressure and temperature. The rate of change in bulk temperature within the control volume can be expressed as

$$\frac{dT_{cv}}{dt} = \frac{1}{\left[m_{cv}\left(\frac{\partial h_{cv}}{\partial T_{cv}}\right)_v - V_{cv}\left(\frac{\partial P_{cv}}{\partial T_{cv}}\right)_v \right]} \times \left\{ \frac{dQ}{dt} + \frac{dm_i}{dt}h_i - \frac{dm_e}{dt}h_e - \frac{dm_{cv}}{dt}h_{cv} - \left[m_{cv}\left(\frac{\partial h_{cv}}{\partial v_{cv}}\right)_T - V_{cv}\left(\frac{\partial P_{cv}}{\partial v_{cv}}\right)_T \right]\left[-\frac{V_{cv}}{m_{cv}^2}\frac{dm_{cv}}{dt} + \frac{1}{m_{cv}}\frac{dV_{cv}}{dt} \right] \right\}. \quad (1)$$

This equation can then be numerically integrated during the simulation to estimate the control volume temperature. In the simulation used here, the real gas equation for R134a that was established by R. Tillner and H. D. Baehr (1993) is used to improve the simulation results because suction superheat might be small in the case of high mass flow rate.

2.2 Volume Equation

It is assumed that the swash plate has a specific angle, a , which is a variable, when the mass flow rate and the operating speed are given as shown in Figure 2. If the variable stoke, l_{var}, is defined to be the difference in piston position between top dead center and bottom dead center, and the angular function of time is $q = 2p\,(rpm/60)\,t$, the volume of the cylinder swept out by the piston motion is expressed as

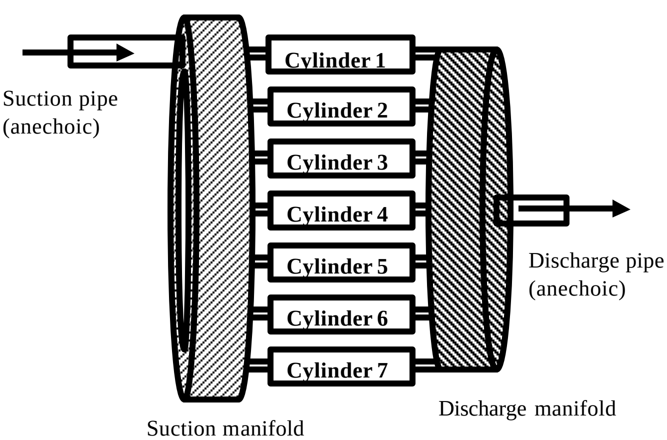

Figure 1: Configuration of a seven-cylinder compressor

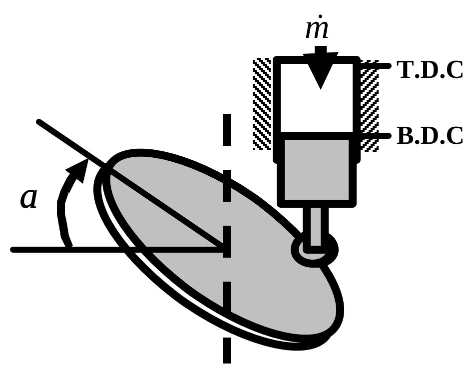

Figure 2: Swash plate cam

$$V_{cv}(t) = V_{cl} + \frac{pD^2}{4}\frac{l_{\text{var}}}{2}\left(1 - \cos(2p\frac{rpm}{60}t)\right). \qquad (2)$$

2.3 Mass Flow Rate

The equation for the mass flow rate through the valve port is derived assuming one-dimensional compressible flow in an orifice. When mass flows through the orifice, the flow may be assumed to be adiabatic because there is neither sufficient time nor a large enough area for any effective heat transfer to take place. Also, it is assumed that there is no work done and that the change in potential energy is very small compared to the changes in enthalpy and internal energy. The mass flow rate is given by

$$\dot{m}_2 = A_2 r_1 C_d Y \sqrt{\frac{2[(p_1 - p_2)/r_1]}{1 - (A_2/A_1)^2}}, \qquad (3)$$

where $Y = \left\{\frac{[k/(k-1)](p_2/p_1)^{2/k}[1-(p_2/p_1)^{(k-1)/k}][1-(A_2/A_1)^2]}{[1-(A_2/A_1)^2(p_2/p_1)^{2/k}][1-p_2/p_1]}\right\}^{1/2}$, which is called the compressibility factor and C_d is the discharge coefficient. The compressibility factor is usually assumed to be a function of area ratio, A_2/A_1, and pressure ratio, p_2/p_1.

2.4 Valve Dynamics

The motion of the valve between its limiting constraints, which are the stopper and the valve seat in the compressor, leads to dynamic contact (i.e., impacts). Because the valve port is fairly symmetrical, the tilting motion of the valve need not be considered. Therefore, the valve motion is approximated as one-dimensional in the transverse direction and a linear elastic beam model, which is clamped at one end and is free at the other end, is used. The beam is rigidly supported at one end and has nonlinear elastic supports along certain sections of its length (Figure 3). The effects of impacts and the pressure difference between the suction manifold and cylinder are considered. The transverse flexural deflection, $w(x,t)$, of the beam at a location, x, at time, t, is governed by the following inhomogeneous differential equation of motion:

$$EI(x)\frac{\partial^4 w}{\partial x^4} + rA(x)\frac{\partial^2 w}{\partial t^2} + S(t)d(x - x^*) = p(x,t). \qquad (4)$$

The motion of the left end of the beam is constrained by the stop. In the model, the stops are represented as piecewise linear springs. At the point of contact, $x = x^*$, the impact force, $S(x^*,t)$, is a function of displacement, $w(x^*,t)$, and can be expressed as

$$S(b,t) = \left\{\begin{array}{cc} k_1\{w(x^*,t) - d_1\} & w(x^*,t) \ge d_1 \\ 0 & -d_2 \le w(x^*,t) \le d_1 \\ k_2\{w(x^*,t) + d_2\} & w(x^*,t) \le -d_2 \end{array}\right\}. \qquad (5)$$

With this assumption, the beam is permitted to penetrate the stops due to the finite stiffness of the stop. The depth of penetration can be adjusted with the spring constants, k_1 and k_2.

2.5 Effective Force Area

From a phenomenological perspective, it can be said that the net force resulting from the pressure difference between the cylinder and the suction manifold causes the motion of the valve. The gap between the reed valve and valve seat allows the gas to flow. It is assumed that the pressure distribution on the suction port may be responsible for a pressure differential that causes the valve to vibrate but the pressure distribution on the area excluding the suction port has a little effect on the vibration of the valve.

The effective force area (EFA) is expressed as follows:

$$\text{EFA} = p \times (r * z)^2. \qquad (6)$$

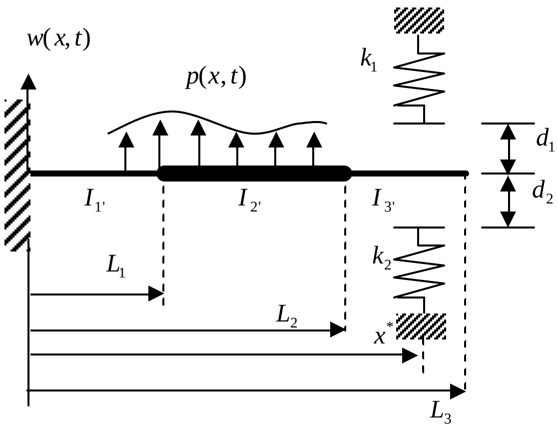

Figure 3: Schematic of the valve with piecewise springs

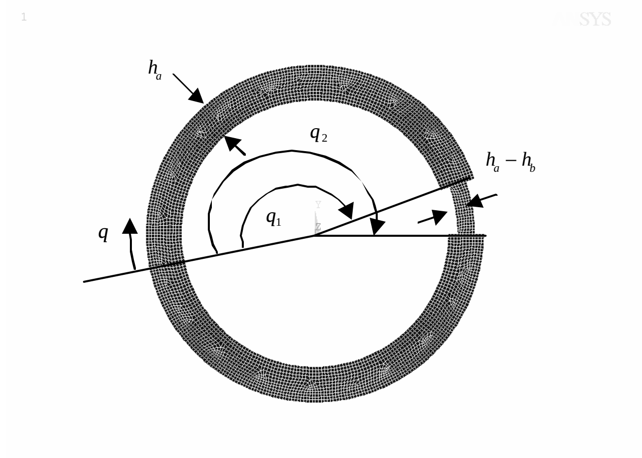

Figure 4: Schematic of a suction manifold

It is thought that the pressure difference between the pressure in the cylinder and that in the suction manifold is affected by the speed of the piston, the period, the mass flow rate and so on. According to several references (Possamai *et al*., 1995, Deschamps *et al*., 1996), the pressure profile reaches a maximum on the suction port area whereas a sharp pressure drop occurs at the outer part of the suction port on which the pressure level never recovers to positive values. Furthermore, it is assumed that the amplitude of the negative value depends on the flow Reynolds number. When the flow Reynolds number increases, the pressure drop increases. The presence of a negative pressure may reduce the resultant forces acting on the valve. A force number, a_1, that describes this reduction is defined as follows:

$$a_1 = \left. \frac{\dot{m}}{\dfrac{\max\_\ stroke}{period}} \right|_{at\,2000rpm\,40kg/h} \Bigg/ \left. \frac{\dot{m}}{\dfrac{\max\_\ stroke}{period}} \right|_{at\ operating\,condition} . \tag{7}$$

The pressure difference between the pressure in the cylinder and that in the suction manifold is multiplied by the force number, which is the ratio of mass per unit piston stroke at 2000 RPM and 40 kg/h to the mass per unit piston stroke at a certain chosen operating condition ($a_1 \times \Delta p$).

2.5 Effective Flow Area

It is assumed that the effective flow area is a simple function of the displacement of the beam as follows:

$$A_2 = pDw(L^*, t) . \tag{8}$$

In this simulation, L^* is assumed to be the distance from the clamped point to the center of the suction port.

2.6 Equation for the Suction Manifold with the Area Change

The real suction manifold is modeled with a simplified annular cavity (Figure 4) considering the area change due to the discharge pipe. The pressure variation along the radial direction in the annular cavity is not included because the length difference between the inner and outer radius is small enough to justify an assumption of uniform pressure distribution along the radial direction (i.e., one dimensional plane wave model).

An analytical method is developed here to predict the natural frequencies of the suction manifold with a widely distributed area change using the Heaviside unit step function. The Rayleigh-Ritz method is used to find the solution of the associated eigenvalue problem. In order to solve the inhomogeneous equation, Galerkin's method is applied. The wave equation for a variable cross-sectional area was derived by Soedel and P. Lai (1996 a, b). The inhomogeneous plane wave equation of a simplified model with non-uniform cross sectional areas for the fluctuating pressure becomes

$$-\frac{h\partial^2 p}{r^2\partial q^2}-\frac{1}{r^2}\frac{\partial h}{\partial q}\frac{\partial p}{\partial q}+\frac{h}{c^2}\frac{\partial^2 p}{\partial t^2}=\frac{\ddot{m}}{rb}d(q-q^*)\,. \tag{9}$$

The width of the suction manifold in the circumferential direction can be represented as

$$h=h_a-h_b H(q-q_1)+h_b H(q-q_2)\,. \tag{10}$$

In order to analyze the annular cavity using the approximate plane wave model, the Rayleigh-Ritz method is used. There is no boundary condition in the q direction because the annular cavity is fully connected; therefore, the following two sets of natural modes are obtained:

$$(p_k)_1=\sin k(q-f_1)\ \text{ and }\ (p_k)_2=-\cos k(q-f_1)\,, \tag{11}$$

where k is the integer that determines the mode of interest. Note that f_1 is an arbitrary phase angle that must be included because an annular cavity with an area change does show a preference for the orientation of its modes. In the case of the annular cavity with the area change as shown in Figure 4, the orientation is determined by the location of the area change (discontinuity). It is worthwhile to note at this point in the analysis that each value of k is associated with two modes with different frequencies because of the area change in the circumferential direction. The approximate solution of the differential equation (9) for a source is obtained using Galerkin's method in the form

$$p(q,t)=\sum_{j=1}^{M} p_{kj}(q)q_j(t)\,. \tag{12}$$

The pressure response for multiple inputs/single output in the annular cavity can be formulated with the transfer function and four pole parameters as explained in the following section.

2.7 Four Pole Parameters and Transfer Function

Four pole parameters are very useful for the analysis of composite acoustic systems and have been widely used to analyze gas pulsations in cavities. The annular cavity with area change can be described using four pole parameters as follows:

$$\begin{bmatrix} Q_{1n} \\ P_{1n} \end{bmatrix}=\begin{bmatrix} A_n & B_n \\ C_n & D_n \end{bmatrix}\begin{bmatrix} Q_{2n} \\ P_{2n} \end{bmatrix}, \tag{13}$$

where A_n, B_n, C_n, D_n are four pole parameters and subscripts 1, 2 indicate valve point and anechoic point, respectively.

According to reference (Snowdon, 1971), the coefficients of the four pole parameters are defined as follows:

$$A_n=\left.\frac{Q_{1n}}{Q_{2n}}\right|_{P_{2n}=0} \qquad B_n=\left.\frac{Q_{1n}}{P_{2n}}\right|_{Q_{2n}=0} \qquad C_n=\left.\frac{P_{1n}}{Q_{2n}}\right|_{P_{2n}=0} \qquad D_n=\left.\frac{P_{1n}}{P_{2n}}\right|_{Q_{2n}=0}\,. \tag{14}$$

Suppose the relation between the pressure at any position inside the cavity and the input/output volume flow rate at the single frequency (Kim and Soedel, 1989, 1990), nw, is given by

$$P_n(q)e^{jnwt}=f_{1n}(q,nw)Q_{1n}e^{jnwt}-f_{2n}(q,nw)Q_{2n}e^{jnwt}\,. \tag{15}$$

The four pole parameters for each input/output can be obtained separately in the following way by substituting equation (15) into equation (14):

$$A_n=\left.\frac{Q_{1n}}{Q_{2n}}\right|_{P_{2n}=0}=\frac{f_{2n}(q_2,nw)}{f_{1n}(q_2,nw)} \qquad\qquad B_n=\left.\frac{Q_{1n}}{P_{2n}}\right|_{Q_{2n}=0}=\frac{1}{f_{1n}(q_2,nw)}$$

$$C_n = \left.\frac{P_{1n}}{Q_{2n}}\right|_{P_{2n}=0} = -f_{2n}(q_1, nw) + \frac{f_{1n}(q_1, nw)}{f_{1n}(q_2, nw)} f_{2n}(q_2, nw) \qquad D_n = \left.\frac{P_{1n}}{P_{2n}}\right|_{Q_{2n}=0} = \frac{f_{1n}(q_1, nw)}{f_{1n}(q_2, nw)}.$$

If an annular cavity is connected to an anechoic pipe and then subjected to a harmonic input volume flow source, the relationship between the volume velocity and the pressure at the entrance of the anechoic pipe is

$$Z_{ex} = \frac{P_{2n}}{Q_{2n}} = \frac{rc}{S_e}, \tag{16}$$

where S_e is the cross section area of the anechoic pipe and Z_{ex} is the impedance. This impedance relationship serves as a boundary condition to the annular manifold model.

When equation (16) is substituted into equation (13), the following expression is obtained:

$$\begin{bmatrix} Q_{1n} \\ P_{1n} \end{bmatrix} = \begin{bmatrix} A_n & B_n \\ C_n & D_n \end{bmatrix} \begin{bmatrix} 1 \\ rc/S_e \end{bmatrix} Q_{2n}. \tag{17}$$

From equation (17), the transfer function between the flow rate at the inlet and the flow rate at a valve port can be obtained:

$$T_{Q_n}(w) = \frac{Q_{1n}}{Q_{2n}} = A_n + Z_{ex} B_n. \tag{18}$$

3. Test and Simulation Results

Experiments are needed to verify the accuracy of any simulation model based on first principles. The test stand shown in Figure 5 was constructed to measure the experimental data for the test compressor over various operating conditions. Pressures in the cylinders, gas pulsations in the suction manifold, and the variable piston strokes, which depend on the operating conditions, were measured. The simulation is initialized at the end of the discharge process. The initial conditions were selected with temperature and pressure, which were measured using a thermo-couple and transducer C in Figure 5. In this simulation, the discharge process is not considered because the gas pulsations in the suction manifold are of primary interest. The analytical results of pressure responses in the cylinder and gas pulsations in the suction manifold are compared with the experimental data.

3.1 Test Stand

Referring to Figure 5, the R-134a gas enters in the suction manifold from the evaporator and feeds through the suction ports and valves into the cylinder. The gas in the cylinder is then compressed by pistons and forced out through the discharge ports and valves into the discharge plenum. The main function of the compressor in the refrigeration cycle is to convert the mechanical energy into thermal energy. After hot gas passes through the condenser, low pressure and temperature gas is created within the expansion valve.

The compressor chosen and installed on the stand was a seven-cylinder reciprocating compressor. The average mass flow rate was measured at the test stand using a flow meter. Because of the difficulty in measuring the transient temperatures in the cylinder, the average temperatures were measured at the suction and discharge lines. Small dynamic transducers were installed in the cylinders and the pressure responses in the cylinders were measured with these transducers. The measured data were recorded and processed using dynamic data processing equipment.

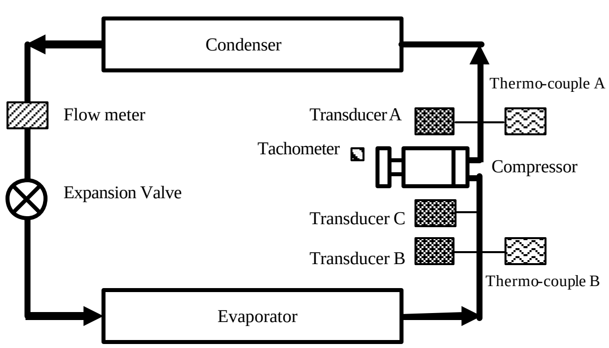

Figure 5: Schematic of experimental test apparatus

The compressor speed, the discharge pressure and the mean mass flow rate were measured using the tachometer, transducer A and the flow meter, which set the conditions for the test. The suction pressure, the discharge gas temperature and the suction gas temperature were measured with transducer B, thermo-couple A and B, respectively, and the suction pressure pulsation was measured with transducer C. The transducers and the thermo-couples were installed near the discharge port and the suction port, respectively.

3.2 Comparison of the Test Results with Simulated Results

Two different operating conditions were selected in order to verify the simulation results. At each condition, pressures in the cylinders and gas pulsations in the suction manifold were measured as a function of time. The pressure response time histories are converted into pressure response volume histories. The pressure response volume histories in the cylinder were estimated in the case of 1500 and 2000 rpm with 60kg/h capacity as shown in Figure 6. The simulated pressure responses are in good agreement with the experimental data. From the results, it is verified that the parameters determined from experiments and simulation are of acceptable accuracy.

Using the mass flow rates calculated, the gas pulsations were calculated at a position rotated 100 degrees from the inlet of the suction pipe. The pressure pulsations using the analytical models in the suction manifold are in good agreement with the experimental results at 1500 rpm and 2000 rpm with 60 kg/h capacity.

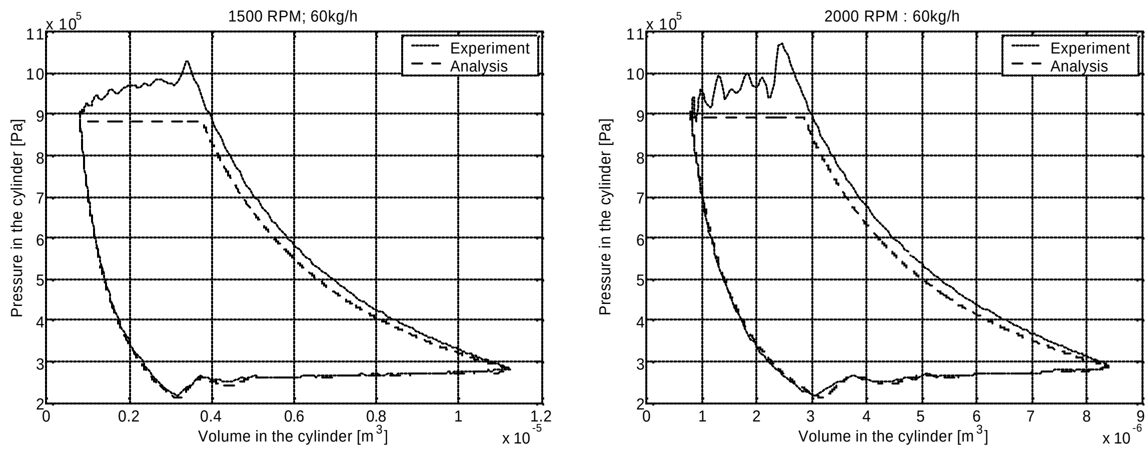

Figure 6: Pressure responses in the cylinder at (a) 1500 rpm and (b) 2000 rpm with 60kg/h capacity

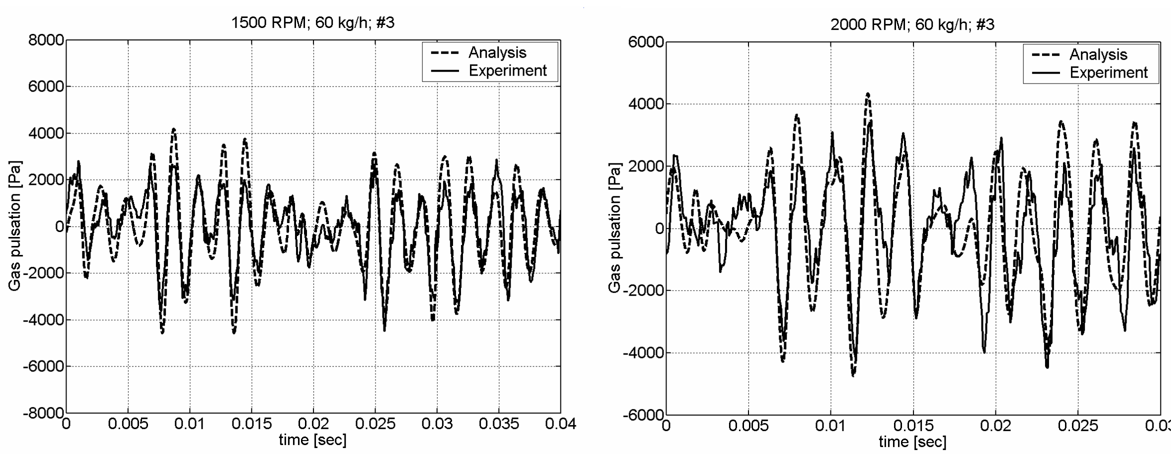

Figure 7: Comparison of the pressure pulsations in the time domain with experimental results at position 3 at (a) 1500 rpm and (b) 2000 rpm with 60kg/h capacity

4. CONCLUSIONS

A simulation model of a seven-cylinder reciprocation compressor was developed to predict the pressure response in the cylinder and the gas pulsations in the suction manifold. The pressure response estimates in the cylinder and gas pulsation estimates in the suction manifold were shown to be in good agreement with experimental data. It was also shown that a characteristic cylinder method for a multi-cylinder compressor model was applicable to predict pressure pulsations in the suction manifold.

NOMENCLATURE

b	height of the suction manifold	(m)
c	speed of sound	(m/s)
D	diameter of the suction port	(m)
h	enthalpy	(kJ/kg)
m	mass	(kg)
P	pressure	(Pa)
Q	volume mass flow rate	(m^3/s)
r	radius of the suction manifold	(m)
rpm	operating speed	(rpm)
t	time	(s)
T	temperature	(K)
V	volume	(m^3)

Subscripts

cv	control volume
cl	clearance

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